Many of my research projects began with coding a simulation or visualization of a mathematical process or concept and discovering interesting conjectures, which in some cases I ended up proving. In this talk I will show a few such simulations, involving families of random combinatorial objects such as Young tableaux, sorting networks, alternating sign matrices, and more. The stories of the rigorous results that resulted from these simulations will provide interesting lessons on the role that experimental mathematics can play in modern math (and specifically probability, combinatorics and related areas) research.